Kernelization for Degree Sequence Completion Problems Using f-Factors

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joint work with

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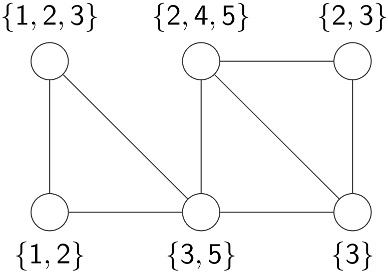
Introductory Example

DEGREE CONSTRAINT EDITING
(Mathieson & Szeider, 2012)



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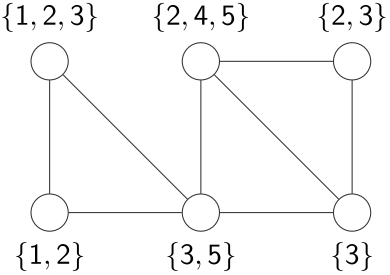

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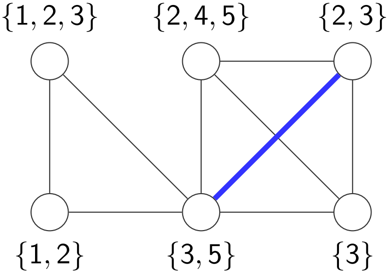

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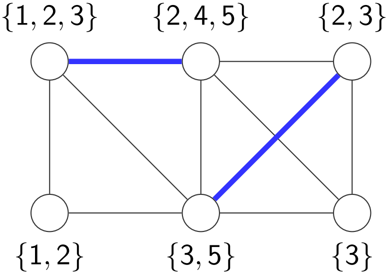

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Input: An undirected graph $G = (V, E)$, two integers $k, r > 0$, and a "degree list function" $\tau\colon V \to 2^{\{0,\ldots,r\}}$.

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- f-Factor: $S = \{e^-\}$, $\tau(v) = \{f(v)\}$ for some $f : V \to \mathbb{N}$

Results

Mathieson & Szeider (2012) introduced (weighted) $\mathrm{DCE}(S)$ and show that it is W[1]-hard wrt. k and FPT wrt. (k, r) for all $\emptyset \neq S \subseteq \{\mathsf{v}^-, \mathsf{e}^+, \mathsf{e}^-\}$.
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- Framework for polynomially bounding parameters of degree sequence completion problems

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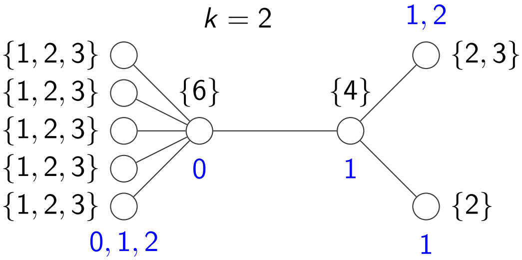

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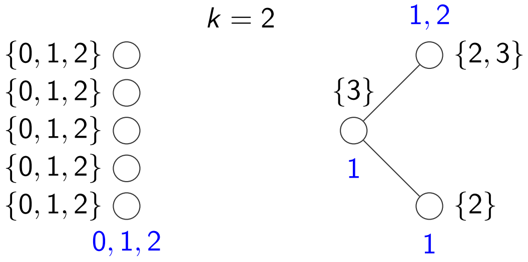

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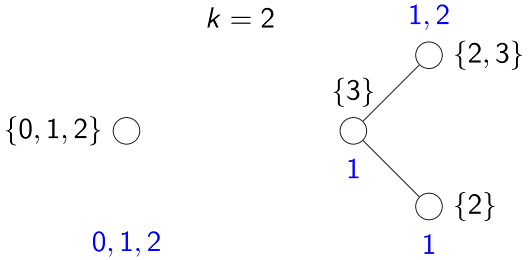

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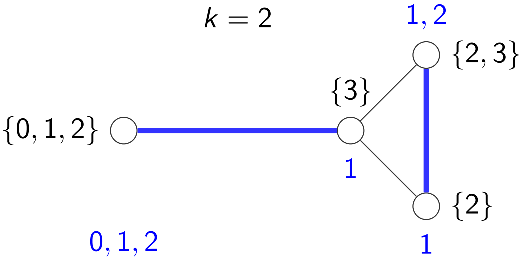

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Correctness follows by some exchange arguments.

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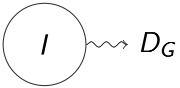

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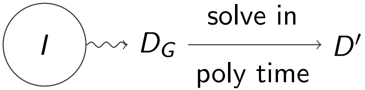

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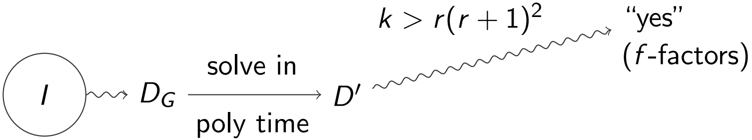

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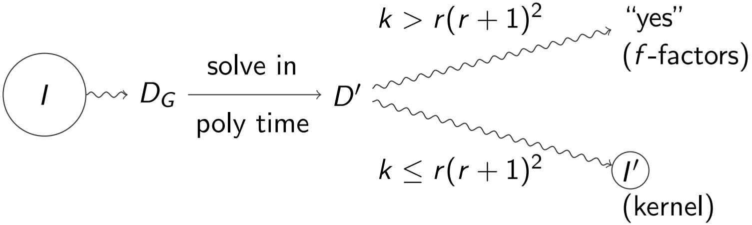

Demands and f-Factors

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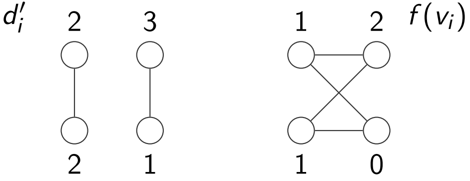

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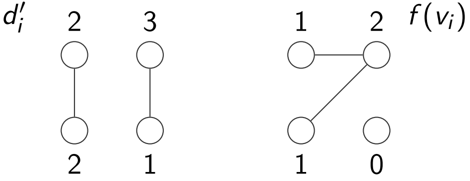

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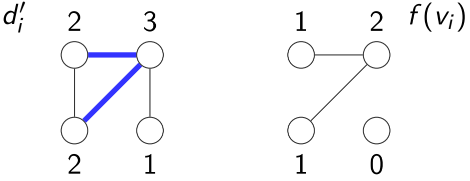

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If there exists a $k' \in \{r(r+1)^2, \ldots, k\}$ *such that*

$$(\deg_G(v_1), \ldots, \deg_G(v_n), 2k', r, \phi)$$

with $\phi(i) := \tau(v_i)$ *is a yes-instance of* NCE, *then* I *is a yes-instance.*

Win-Win Kernel

Theorem

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Case 2. $k \leq r(r+1)^2$:
Simply run the previous $O(kr^2)$-vertex kernelization on I. □

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Let Π be some tuple property (e.g. regularity $\forall i \neq j : d_i = d_j$).

Π-DEGREE SEQUENCE COMPLETION (Π-DSC)
Input: A graph $G = (V, E)$, an integer $k \in \mathbb{N}$.
Question: Is there a set of edges $E' \subseteq \binom{V}{2} \setminus E$ with $|E'| \leq k$ such that the degree sequence of $G' = (V, E \cup E')$ fulfills Π?

Results for Π-Degree Sequence Completion

$\Delta_G / \Delta_{G'}$: maximum input/output degree ($\Delta_{G'} \leq \Delta_G + k$.)

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If deciding Π is fixed-parameter tractable with respect to the largest number in the tuple, then Π-DSC *is fixed-parameter tractable (weak kernel) with respect to* (k, Δ_G).

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If Π-NSC *is polynomial-time solvable and* Π-DSC *admits a (polynomial) kernel with respect to* (k, Δ_G), *then* Π-DSC *admits a (polynomial) kernel with respect to* $\Delta_{G'}$.

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Example Application:

Π $\hat{=}$ Degree Anonymity (Hartung et al., 2013).

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- Kernel transformation by solving a simpler number editing problem and translating the solution back into a solution for the graph problem using f-factors.
- Generalized framework separating “problem-specific” parts from “universal” parts to transform multivariate problem kernels for degree sequence completion problems into univariate problem kernels.

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Thank You!